The Solution to the Factorial Analysis of Variance

As shown in the Excel file, Howell 13-2, the ANOVA analysis (in the ToolPac) yielded the following table:

Anova: Two-Factor With Replication

SUMMARY	Counting	Rhyming	Adjetive	Imagery	Total
Old					
Count	10	10	10	10	40
Sum	70	69	110	134	383
Average	7.000	6.900	11.000	13.400	9.575
Variance	3.333	4.544	6.222	20.267	15.738
Young					
Count	10	10	10	10	40
Sum	65	76	148	176	465
Average	6.500	7.600	14.800	17.600	11.625
Variance	2.056	3.822	12.178	6.711	28.343
Total					
Count	20	20	20	20	
Sum	135	145	258	310	
Average	6.750	7.250	12.900	15.500	
Variance	2.618	4.092	12.516	17.421	

ANOVA

ce of Varia	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Sample	84.050	1	84.050	11.371	0.001	3.974
Columns	1106.900	3	368.967	49.917	0.000	2.732
Interactior	80.050	3	26.683	3.610	0.017	2.732
Within	532.200	72	7.392			
Total	1803.200	79				

All effects—Sample (Group), Columns (Condition), and the Group x Condition interaction—were statistically significant at $p < .05$. (I provide a sample write-up later.)

For the next analysis I used VassarStats > Two-Way Factorial ANOVA for Independent Samples. Following the directions given there, I obtained the following. (Note that the ANOVA summary table is the same as that for the Excel example.)

Data Entered

	Col 1	Col 2	Col 3	Col 4
Row 1	9 8 6 8 10 4	7 9 6 6 6 11	11 13 8 6 14 11	12 11 16 11 9 23

	6 5 7 7	6 3 8 7	13 13 10 11	12 10 19 11
Row 2	8 6 4 6 7 6 5 7 9 7	10 7 8 10 4 7 10 6 7 7	14 11 18 14 13 22 17 16 12 11	20 16 16 15 18 16 20 22 14 19
Row 3	---	---	---	---
Row 4	---	---	---	---

Summary Data	Within each box: Item 1 = N Item 2 = $\sum X$ Item 3 = Mean Item 4 = $\sum X^2$ Item 5 = Variance Item 6 = Std. Dev. Item 7 = Std. Err.				
	C1	**C2**	**C3**	**C4**	**Tot.**
R1	10 70 7 520 3.33 1.83 0.58	10 69 6.9 517 4.54 2.13 0.67	10 110 11 1266 6.22 2.49 0.79	10 134 13.4 1978 20.27 4.5 1.42	40 383 9.575 4281 15.74 3.97 0.63
R2	10 65 6.5 441 2.06 1.43 0.45	10 76 7.6 612 3.82 1.96 0.62	10 148 14.8 2300 12.18 3.49 1.1	10 176 17.6 3158 6.71 2.59 0.82	40 465 11.625 6511 28.34 5.32 0.84
R3	---	---	---	---	---
R4	---	---	---	---	---
Tot.	20 135 6.75 961 2.62 1.62 0.36	20 145 7.25 1129 4.09 2.02 0.45	20 258 12.9 3566 12.52 3.54 0.79	20 310 15.5 5136 17.42 4.17 0.93	80 848 10.6 10792 22.83 4.78 0.53

ANOVA Summary					
Source	**SS**	**df**	**MS**	**F**	**P**
Rows	84.05	1	84.05	11.37	0.0012
Columns	1106.9	3	368.97	49.92	<.0001
r x c	80.05	3	26.68	3.61	0.0173
Error	532.2	72	7.39		

Total	1803.2	79

Critical Values for the Tukey HSD Test			
	HSD[.05]	**HSD[.01]**	HSD=the absolute [unsigned] difference between any two means (row means, column means, or cell means) required for significance at the designated level: HSD[.05] for the .05 level; HSD[.01] for the .01 level. The HSD test between row means can be meaningfully performed only if the row effect is significant; between column means, only if the column effect is significant; and between cell means, only if the interaction effect is significant.
Rows [2]	1.21	1.61	
Columns [4]	2.26	2.78	
Cells [8]	3.8	4.48	

Since there are only two groups, the HSD test for Rows is unimportant. The *F* test for the row effect was already shown to be significant in the ANOVA Summary table. The significant *F* for the columns effect, however, does require the post hoc HSD test.

From the third row of the Tot subsection of the Summary Data table, the means of the four conditions are 6.75, 7.25, 12.9, and 15.5. From the Tukey HSD table we see that to be statistically significant at $p<.05$ we need a difference between any two means to be at least 2.26 units. The difference between means for Condition 1 and Condition 2 does not satisfy this criterion. Hence we find no evidence that rhyming was any more effective than counting. On the other hand the differences between Condition 3 and Condition 2 (an, of course, Condition 1) was statistically significant. The same can be said about Condition 4 (vs Conditions 1 & 2). Also, the difference between Condition 4 and Condition 3 is statistically significant.

There is still a problem that needs to be addressed, however. The presences of a significant interaction tells us that there is a differential effect of Conditions depending upon which group, young or old, is examined. I will treat this in the next analysis.

In the final analysis, I used SPSS to analyze the data in Howell Table 13-2. The syntax I used for this analysis, assuming the data have been obtained from the Excel worksheet, List, was

SPSS Syntax for Analyzing Howell's Table 13-2

```
*The commands for this first analysis specify an overall 2x3 ANOVA.

DATASET ACTIVATE DataSet1.
UNIANOVA Score BY Group Condition
  /METHOD=SSTYPE(3)
  /INTERCEPT=INCLUDE
  /PLOT=PROFILE(Condition*Group)
  /CRITERIA=ALPHA(0.05)
  /DESIGN=Group Condition Group*Condition.

*Since the first analysis yielded a significant Group (or Age) by Condition Interaction, we should perform simple effects analyses within groups.
This command splits the data file by Group.

SPLIT FILE LAYERED BY Group.
```

*The next set of commands re-run the earlier analysis, only this time the analysis is computed once for each group. Additionally, for each analysis, SPSS is instructed to generate plots of means.

```
UNIANOVA Score BY Group Condition
  /METHOD=SSTYPE(3)
  /INTERCEPT=INCLUDE
  /POSTHOC=Condition(TUKEY)
  /PLOT=PROFILE(Condition*Group)
  /CRITERIA=ALPHA(0.05)
  /DESIGN=Group Condition Group*Condition.
```

You can view the results for this analysis by copying the syntax above, pasting it into a new SPSS syntax file, then running the syntax.

In the Output Navigator (the pane to the left) you can scroll down to a heading labeled, Tests of Between-Subjects Effects to display the following table.

As is the case with many general statists packages, this table has more information than what is typically given in an ANOVA summary table. For instance, we typically are not interested in the [Mean] Corrected Model or Intercept sources of variance since these

Tests of Between-Subjects Effects

Dependent Variable: Score

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1271.000[a]	7	181.571	24.564	.000
Intercept	8988.800	1	8988.800	1216.072	.000
Group	84.050	1	84.050	11.371	.001
Condition	1106.900	3	368.967	49.917	.000
Group * Condition	80.050	3	26.683	3.610	.017
Error	532.200	72	7.392		
Total	10792.000	80			
Corrected Total	1803.200	79			

a. R Squared = .705 (Adjusted R Squared = .676)

typically are always highly significant. Moreover, we usually DO want to report the (Mean) Corrected Total Variance rather than the Total Variance. By double clicking on the table in SPSS the table can be edited so that it looks like the following:

Tests of Between-Subjects Effects

Dependent Variable: Score

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Group	84.050	1	84.050	11.371	.001
Condition	1106.900	3	368.967	49.917	.000
Group * Condition	80.050	3	26.683	3.610	.017
Error	532.200	72	7.392		
Corrected Total	1803.200	79			

As mentioned earlier, since the test for the Group x Condition was significant, the significant effect due to Conditions is different for each Group. Because of this, we would normally be motivated to compute a simple effects analysis for each Group. Computing simple effect analyses is tantamount to computing a separate ONEWAY ANOVA on Condition for each Group. The simple effects syntax, when run, yields the following ANOVA summary table (Note that there are two Anovas given in the table).

Tests of Between-Subjects Effects

Dependent Variable: Score

Group	Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Old	Corrected Model	304.475[a]	3	101.492	11.813	.000
	Intercept	3667.225	1	3667.225	426.835	.000
	Group	.000	0	.	.	.
	Condition	304.475	3	101.492	11.813	.000
	Group * Condition	.000	0	.	.	.
	Error	309.300	36	8.592		
	Total	4281.000	40			
	Corrected Total	613.775	39			
Young	Corrected Model	882.475[b]	3	294.158	47.509	.000
	Intercept	5405.625	1	5405.625	873.048	.000
	Group	.000	0	.	.	.
	Condition	882.475	3	294.158	47.509	.000
	Group * Condition	.000	0	.	.	.
	Error	222.900	36	6.192		
	Total	6511.000	40			
	Corrected Total	1105.375	39			

a. R Squared = .496 (Adjusted R Squared = .454)

b. R Squared = .798 (Adjusted R Squared = .782)

Again, there is more information given in the table than we need. In addition to the Corrected Model, Intercept, and Total sources of variance, we do not need the Group and Group * Condition source of variance since, in simple effects, these are not factors. By double clicking on this table in SPSS, the table can be edited to look like the following.

Simple Effect Tests of Between-Subjects Effects

Dependent Variable: Score

Group	Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Old	Condition	304.475	3	101.492	11.813	.000
	Error	309.300	36	8.592		
	Corrected Total	613.775	39			
Young	Condition	882.475	3	294.158	47.509	.000
	Error	222.900	36	6.192		
	Corrected Total	1105.375	39			

The Condition effect is significant within both groups.

The syntax also instructed SPSS to compute post hoc comparisons, using Tukey's HSD test. Since the data file was split on Group, these comparisons are performed separately for each group. The post hoc analyses are summarized in a homogeneous subsets table:

Group=Old

Tukey HSD[a,b]

Condition	N	Subset 1	Subset 2
Rhyming	10	6.90	
Counting	10	7.00	
Adjective	10		11.00
Imagery	10		13.40
Sig.		1.000	.276

Means for groups in homogeneous subsets are displayed.
Based on observed means.
The error term is Mean Square(Error) = 8.592.

a. Uses Harmonic Mean Sample Size = 10.000.

b. Alpha = 0.05.

Group=Young

Tukey HSD[a,b]

Condition	N	Subset 1	Subset 2
Counting	10	6.50	
Rhyming	10	7.60	
Adjective	10		14.80
Imagery	10		17.60
Sig.		.757	.074

Means for groups in homogeneous subsets are displayed.
Based on observed means.
The error term is Mean Square(Error) = 6.192.

a. Uses Harmonic Mean Sample Size = 10.000.

b. Alpha = 0.05.

For both age groups, the Adjective and Imagery conditions resulted in higher levels of recall than did the Rhyming and Counting conditions. Furthermore, the difference was greater in the Young Group. This was evident in the plots from the over all analysis: