In this brief paper, I show how the total sums of squares (SS) for variable, \( Y_{ij} \) can be partitioned into two sources, sums of squares between groups (SS\(_B\)) and sums of squares within groups (SS\(_W\)).

We begin with deviation scores, \((Y_{ij} - \bar{Y}_j)\), where \( Y_{ij} \) is the score for an individual, \( i \), in group \( j \). Next, square and sum the deviation scores over all individuals, over all groups:

\[
\sum_{j} \sum_{i} (Y_{ij} - \bar{Y}_j)^2 = \sum_{j} \sum_{i} [(Y_{ij} - \bar{Y}_j) + (\bar{Y}_j - \bar{Y}_n)]^2
\]

\[
= \sum_{j} \sum_{i} (Y_{ij} - \bar{Y}_j)^2 + \sum_{j} \sum_{i} (\bar{Y}_j - \bar{Y}_n)^2
\]

\[
+ 2 \sum_{j} \sum_{i} (Y_{ij} - \bar{Y}_j)(\bar{Y}_j - \bar{Y}_n)
\]

Note, however, that

\[
2 \sum_{j} \sum_{i} (Y_{ij} - \bar{Y}_j)(\bar{Y}_j - \bar{Y}_n) = 2 \sum_{j} (\bar{Y}_j - \bar{Y}_n) \sum_{i} (Y_{ij} - \bar{Y}_j)
\]

since \((Y_{ij} - \bar{Y}_j)\) is the same for all individuals, \( i \), in group \( j \).

It might be easier to consider the expression,

\[
2 \sum_{j} \sum_{i} (y_{ij})(\bar{Y}_j) = 2 \sum_{j} (\bar{Y}_j) \sum_{i} (y_{ij})
\]

where \( y_{ij} = (Y_{ij} - \bar{Y}_j) \) is the deviation score for individuals in Group \( j \), and \( \bar{Y}_j \) is the deviation of \( \bar{Y} \) for Group \( j \) from the over all mean.

Furthermore,

\[
\sum_{j} \sum_{i} (\bar{Y}_j - \bar{Y}_n)^2 = \sum_{j} n_j (\bar{Y}_j - \bar{Y}_n)^2
\]

because, within each group, \( j \), \((\bar{Y}_j - \bar{Y}_n)\) is constant (i.e., \( \sum c = nc \).)
Hence,

$$
\sum_j \sum_i (Y_{ij} - \bar{Y}_j)^2 = \sum_j \sum_i (Y_{ij} - \bar{Y}_j)^2 + \sum_j n_j (\bar{Y}_{ij} - \bar{Y}_j)^2.
$$

The first term on the right, $\sum_j \sum_i (Y_{ij} - \bar{Y}_j)^2$, is called the *within group* sums of squares (it is the sum of squared deviations of all individuals from the individual’s respective group mean.) The second term on the right, $\sum_j n_j (\bar{Y}_{ij} - \bar{Y}_j)^2$, is called the *between groups* sums of squares (it is equal to the sum of squared deviations of each group mean from the overall mean.)