Introduction to Path Analysis
Path Coefficient $X_1$ to $X_4 (p_{41})$
Path Coefficient $X_1$ to $X_5$ ($p_{51}$)
The Rest of the Path Coefficients
Disturbance Terms (Residuals)
Exogenous Variables (X₁, X₂, X₃)
Endogenous Variables \((X_4, X_5, X_6)\) Note, \(X_4\) and \(X_5\) are exogenous to \(X_6\)
Structural Equation for $X_4$: $X_4 = p_{41}X_1 +$
Structural Equation for $X_4$: $X_4 = p_{41}X_1 + p_{42}X_2$
Structural Equation for $X_4$: $X_4 = p_{41}X_1 + p_{42}X_2 + p_{43}X_3$
Structural Equation for $X_4$: $X_4 = p_{41}X_1 + p_{42}X_2 + p_{43}X_3 + \epsilon_4$
Structural Equation for $X_5$: $X_5 = p_{51}X_1 + p_{52}X_2 + p_{53}X_3 + e_5$
Structural Equation for $X_6$: $X_5 = p_{61}X_1 + p_{62}X_2 + p_{63}X_3 + p_{64}X_4 + ...$
Structural Equation for $X_6$: $X_6 = p_{61}X_1 + p_{62}X_2 + p_{63}X_3 + p_{64}X_4 + p_{65}(p_{51}X_1 + p_{52}X_2 + p_{53}X_3 + p_{54}X_4) + e_6$
The correlation between $X_1$ and $X_4$: $\rho_{14} = p_{41} + p_{42} \rho_{12} + p_{43} \rho_{13}$
The Correlation Between X1 and X5: $\rho_{15} = p_{51} + p_{52} \rho_{12} + p_{53} \rho_{13} + p_{54} \rho_{14}$
The equation on the previous slide can be expanded

Recalling that the correlation between $X_1$ and $X_4$ is,

$$\rho_{14} = p_{14} + p_{42}\rho_{12} + p_{43}\rho_{13},$$

and substituting this equation into the one given on the previous slide, yields

$$\rho_{15} = p_{51} + p_{52}\rho_{12} + p_{53}\rho_{13} + p_{54}\rho_{14}$$

{From the previous slide}

$$= p_{51} + p_{52}\rho_{12} + p_{53}\rho_{13} + p_{54}(p_{41} + p_{42}\rho_{12} + p_{43}\rho_{13})$$

$$= p_{51} + p_{54}p_{41} + p_{52}\rho_{12} + p_{53}\rho_{13} + p_{54}p_{42}\rho_{12} + p_{54}p_{43}\rho_{13}.$$  

The two highlighted terms ($p_{51}$ and $p_{54}p_{41}$) are, respectively, the *direct* effect of $X_1$ on $X_5$ and the *indirect* effect of $X_1$ on $X_5$ operating through $X_4$.

The remaining terms ($p_{52}\rho_{12}$, $p_{53}\rho_{13}$, $p_{54}p_{42}\rho_{12}$, and $p_{54}p_{43}\rho_{13}$) are non-causal effects (with respect to the casual effect of $X_1$ on $X_5$, and represent *direct* effects of $X_2$ and $X_3$ on $X_5$ ($p_{52}\rho_{12}$ and $p_{53}\rho_{13}$) and *indirect* effects ($p_{54}p_{42}\rho_{12}$ and $p_{54}p_{43}\rho_{13}$).
Direct effect of $X_1$ on $X_5$; Indirect effect of $X_1$ on $X_5$ via $X_4$
We could follow similar steps to compute the correlation between X1 and X6:

$$r_{16} = p_{61} + p_{41}p_{64} + p_{51}p_{65} + p_{41}p_{54}p_{65} + (r_{16} - [p_{61} + p_{41}p_{64} + p_{51}p_{65} + p_{41}p_{54}p_{65}])$$

where the highlighted terms are the direct and indirect effects. The last term, in parentheses, on the right is the residual, or spurious, correlation between X₁ and X₆.

The direct and indirect effects are shown on the next slide.
Direct effect of $X_1$ on $X_6$; Indirect effects of $X_1$ on $X_5$ via $X_4$ and $X_5$. 
Similarly, we could show the decomposition of direct and indirect effects for $X_1$, $X_2$, $X_3$, $X_4$ and $X_5$ on $X_6$:

$r_{26} = p_{62} + p_{42}p_{64} + p_{52}p_{65} + p_{42}p_{54}p_{65} + (r_{26} - [p_{62} + p_{42}p_{64} + p_{52}p_{65} + p_{42}p_{54}p_{65}])$

$r_{36} = p_{63} + p_{43}p_{64} + p_{53}p_{65} + p_{43}p_{54}p_{65} + (r_{26} - [p_{63} + p_{43}p_{64} + p_{53}p_{65} + p_{43}p_{54}p_{65}])$

$r_{46} = p_{64} + p_{54}p_{65} + (r_{26} - [p_{64} + p_{54}p_{65}])$

$r_{56} = p_{65} + (r_{26} - p_{65})$

Direct and indirect effects are highlighted.
Computing the path coefficients ($p_i$).

Path coefficients can be computed using multiple linear regression. The coefficients are identical to the standardized partial regression weights, i.e., the $\beta$s.

To set-up a regression analysis for the model portrayed here, compute three regression equations:

\[
\hat{X}_4 = \beta_{41} X_1 + \beta_{42} X_2 + \beta_{43} X_3 \\
\hat{X}_5 = \beta_{51} X_1 + \beta_{52} X_2 + \beta_{53} X_3 + \beta_{54} X_4 \\
\hat{X}_6 = \beta_{61} X_1 + \beta_{62} X_2 + \beta_{63} X_3 + \beta_{64} X_4 + \beta_{65} X_5
\]

Since, in path analysis, the path coefficients are equivalent to the standardized regression coefficients (i.e., the $\beta$s), we have,

\[
\hat{X}_4 = p_{41} X_1 + p_{42} X_2 + p_{43} X_3 \\
\hat{X}_5 = p_{51} X_1 + p_{52} X_2 + p_{53} X_3 + p_{54} X_4 \\
\hat{X}_6 = p_{61} X_1 + p_{62} X_2 + p_{63} X_3 + p_{64} X_4 + p_{65} X_5
\]
A model similar to Pajares & Miller

- HS Math
- Math Anxiety
- Math SE
- Math SC
- Math Performance
- College Math
- Math Usefulness

SEX
To test this model, estimate the following regression equations:

\[ HSmath = \beta_{21}(SEX) \]
\[ COLmath = \beta_{31}(SEX) + \beta_{32}(HSmath) \]
\[ MathSE = \beta_{41}(SEX) + \beta_{42}(HSmath) + \beta_{43}(COLmath) \]
\[ MathSC = \beta_{51}(SEX) + \beta_{53}(COLmath) \]
\[ MathAnx = \beta_{62}(HSmath) + \beta_{64}(MathSE) \]
\[ MathUse = \beta_{73}(COLmath) \]
\[ MathPerf = \beta_{86}(MathAnx) + \beta_{85}(MathSC) + \beta_{84}(MathSE) \]
Computing the regressions yields the standardized regression coefficients ($\beta$s), which are the desired path coefficients.

The following slide incorporates the path coefficients.
The reduce model (after removing non-significant paths)
The next step in the analysis would be to compute a new set of regression equations to obtain the path coefficients for the remaining paths.

Perhaps we will have time to do this in class.