Statistics Tutorial: F Distribution

The ***f* statistic**, also known as an ***f* value**, is a [random variable](http://stattrek.com/Help/Glossary.aspx?Target=Random_variable) that has an F distribution. (We discuss the F distribution in the next section.)

Here are the steps required to compute an ***f* statistic**:

* Select a random sample of size *n*1 from a normal population, having a standard deviation equal to σ1.
* Select an independent random sample of size *n*2 from a normal population, having a standard deviation equal to σ2.
* The *f* statistic is the ratio of *s*12/σ12 and *s*22/σ22.

The following equivalent equations are commonly used to compute an *f* statistic:

*f* = [ *s*12/σ12 ] / [ *s*22/σ22 ]   
*f* = [ *s*12 \* σ22 ] / [ *s*22 \* σ12 ]   
*f* = [ Χ21 / *v*1 ] / [ Χ22 / *v*2 ]   
*f* = [ Χ21 \* *v*2 ] / [ Χ22 \* *v*1 ]

where σ1 is the standard deviation of population 1, *s*1 is the standard deviation of the sample drawn from population 1, σ2 is the standard deviation of population 2, *s*2 is the standard deviation of the sample drawn from population 2, Χ21 is the [chi-square statistic](http://stattrek.com/Help/Glossary.aspx?Target=Chi_square_statistic) for the sample drawn from population 1, *v*1 is the [degrees of freedom](http://stattrek.com/Help/Glossary.aspx?Target=Degrees_of_freedom) for Χ21, Χ22 is the chi-square statistic for the sample drawn from population 2, and *v*2 is the degrees of freedom for Χ22 . Note that degrees of freedom *v*1 = *n*1 - 1, and degrees of freedom *v*2 = *n*2 - 1 .

The F Distribution

The distribution of all possible values of the *f* statistic is called an **F distribution**, with *v*1 = *n*1 - 1 and *v*2 = *n*2 - 1 degrees of freedom.

The curve of the F distribution depends on the degrees of freedom, *v*1 and *v*2. When describing an F distribution, the number of degrees of freedom associated with the standard deviation in the numerator of the *f* statistic is always stated first. Thus, *f*(5, 9) would refer to an F distribution with *v*1 = 5 and *v*2 = 9 degrees of freedom; whereas *f*(9, 5) would refer to an F distribution with *v*1 = 9 and *v*2 = 5 degrees of freedom. Note that the curve represented by *f*(5, 9) would differ from the curve represented by *f*(9, 5).

The F distribution has the following properties:

* The mean of the distribution is equal to *v*2 / ( *v*2 - 2 ) for *v*2 > 2.
* The [variance](http://stattrek.com/Help/Glossary.aspx?Target=Variance) is equal to [ 2 \* *v*22 \* ( *v*1 + *v*1 - 2 ) ] / [ *v*1 \* ( *v*2 - 2 )2 \* ( *v*2 - 4 ) ] for *v*2 > 4.

Cumulative Probability and the F Distribution

Every *f* statistic can be associated with a unique [cumulative probability](http://stattrek.com/Help/Glossary.aspx?Target=Cumulative_probability). This cumulative probability represents the likelihood that the *f* statistic is less than or equal to a specified value.

Statisticians use *f*α to represent the value of an *f* statistic having a cumulative probability of (1 - α). For example, suppose we were interested in the *f* statistic having a cumulative probability of 0.95. We would refer to that *f* statistic as f0.05, since (1 - 0.95) = 0.05.

Of course, to find the value of *f*α, we would need to know the degrees of freedom, *v*1 and *v*2. Notationally, the degrees of freedom appear in parentheses as follows: *f*α(*v*1,*v*2). Thus, *f*0.05(5, 7) refers to value of the f statistic having a cumulative probability of 0.95, *v*1 = 5 degrees of freedom, and *v*2 = 7 degrees of freedom.

The easiest way to find the value of a particular *f* statistic is to use the [F Distribution Calculator](http://stattrek.com/Tables/F.aspx), a free tool provided by Stat Trek. For example, the value of *f*0.05(5, 7) is 3.97. The use of the F Distribution Calculator is illustrated in the examples below.

F Distribution Calculator

The F Distribution Calculator solves common statistics problems, based on the F distribution. The calculator computes cumulative probabilities, based on simple inputs. Clear instructions guide you to an accurate solution, quickly and easily. If anything is unclear, frequently-asked questions and sample problems provide straightforward explanations. The calculator is free. It can be found under the Stat Tables tab, which appears in the header of every Stat Trek web page.

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| [F Distribution Calculator](http://stattrek.com/Tables/F.aspx) |

Sample Problems

**Example 1**  
  
Suppose you randomly select 7 women from a population of women, and 12 men from a population of men. The table below shows the standard deviation in each sample and in each population.

|  |  |  |
| --- | --- | --- |
| **Population** | **Population standard deviation** | **Sample standard deviation** |
| Women | 30 | 35 |
| Men | 50 | 45 |

Compute the f statistic.

*Solution A:* The f statistic can be computed from the population and sample standard deviations, using the following equation:

*f* = [ *s*12/σ12 ] / [ *s*22/σ22 ]

where σ1 is the standard deviation of population 1, *s*1 is the standard deviation of the sample drawn from population 1, σ2 is the standard deviation of population 2, and *s*1 is the standard deviation of the sample drawn from population 2.

As you can see from the equation, there are actually two ways to compute an f statistic from these data. If the women's data appears in the numerator, we can calculate an f statistic as follows:

*f* = ( 352 / 302 ) / ( 452 / 502 ) = (1225 / 900) / (2025 / 2500) = 1.361 / 0.81 = 1.68

For this calculation, the numerator degrees of freedom *v*1 are 7 - 1 or 6; and the denominator degrees of freedom *v*2 are 12 - 1 or 11.

On the other hand, if the men's data appears in the numerator, we can calculate an f statistic as follows:

*f* = ( 452 / 502 ) / ( 352 / 302 ) = (2025 / 2500) / (1225 / 900) = 0.81 / 1.361 = 0.595

For this calculation, the numerator degrees of freedom *v*1 are 12 - 1 or 11; and the denominator degrees of freedom *v*2 are 7 - 1 or 6.

When you are trying to find the cumulative probability associated with an f statistic, you need to know *v*1 and *v*2. This point is illustrated in the next example.

**Example 2**  
  
Find the cumulative probability associated with each of the f statistics from Example 1, above.

*Solution:* To solve this problem, we need to find the degrees of freedom for each sample. Then, we will use the [F Distribution Calculator](http://stattrek.com/Tables/F.aspx) to find the probabilities.

* The degrees of freedom for the sample of women is equal to *n* - 1 = 7 - 1 = 6.
* The degrees of freedom for the sample of men is equal to *n* - 1 = 12 - 1 = 11.

Therefore, when the women's data appear in the numerator, the numerator degrees of freedom *v*1 is equal to 6; and the denominator degrees of freedom *v*2 is equal to 11. And, based on the computations shown in the previous example, the f statistic is equal to 1.68. We plug these values into the F Distribution Calculator and find that the cumulative probability is 0.78.

On the other hand, when the men's data appear in the numerator, the numerator degrees of freedom *v*1 is equal to 11; and the denominator degrees of freedom *v*2 is equal to 6. And, based on the computations shown in the previous example, the f statistic is equal to 0.595. We plug these values into the F Distribution Calculator and find that the cumulative probability is 0.22.